Quantum World

2026

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Miscellaneous Notes [1]

Contents

1. Lecture 1: EPR and all that

20.04

- Einstein did not care as much about determinism as his "God does not play dice" might make you think.

2. Lecture 2: Measurement

22.04

- Polaroid filters experiment
- "Collapse" happens through Projection operators.
- When does collapse happen? At measurement or at the filters

3. Lecture 3

4. Lecture 4

5. Lecture 5A: Contextuality

18.05.2025

If there exists a joint probability distribution $P(A, B, C)$, then $P(A, B) = \sum_C P(A, B, C)$

If $P(A, B)$, $P(B, C)$ and $P(A, C)$ come from different *urns*, with three different joint distributions $P(A, B, C)$, then there does not exist a single joint probability distribution that can explain the marginals.

contextual a set of probability distributions is *contextual* if they cannot be expressed as marginals of a joint distribution

non-contextual a set of probability distributions is *non-contextual* if there is no such joint probability distribution

For a given set of marginals, *how do we establish* the existence of a joint distribution?

5.1. Contextuality implies the Bell Inequalities

Let us **assume** non-contextuality of the marginals (i.e assume a joint exists):

This allows us to speak of probabilities involving A,B,and C:

$$p(A \neq B) = p(B \neq C \land C = A) + p(B = C \land C \neq A) \quad (1)$$

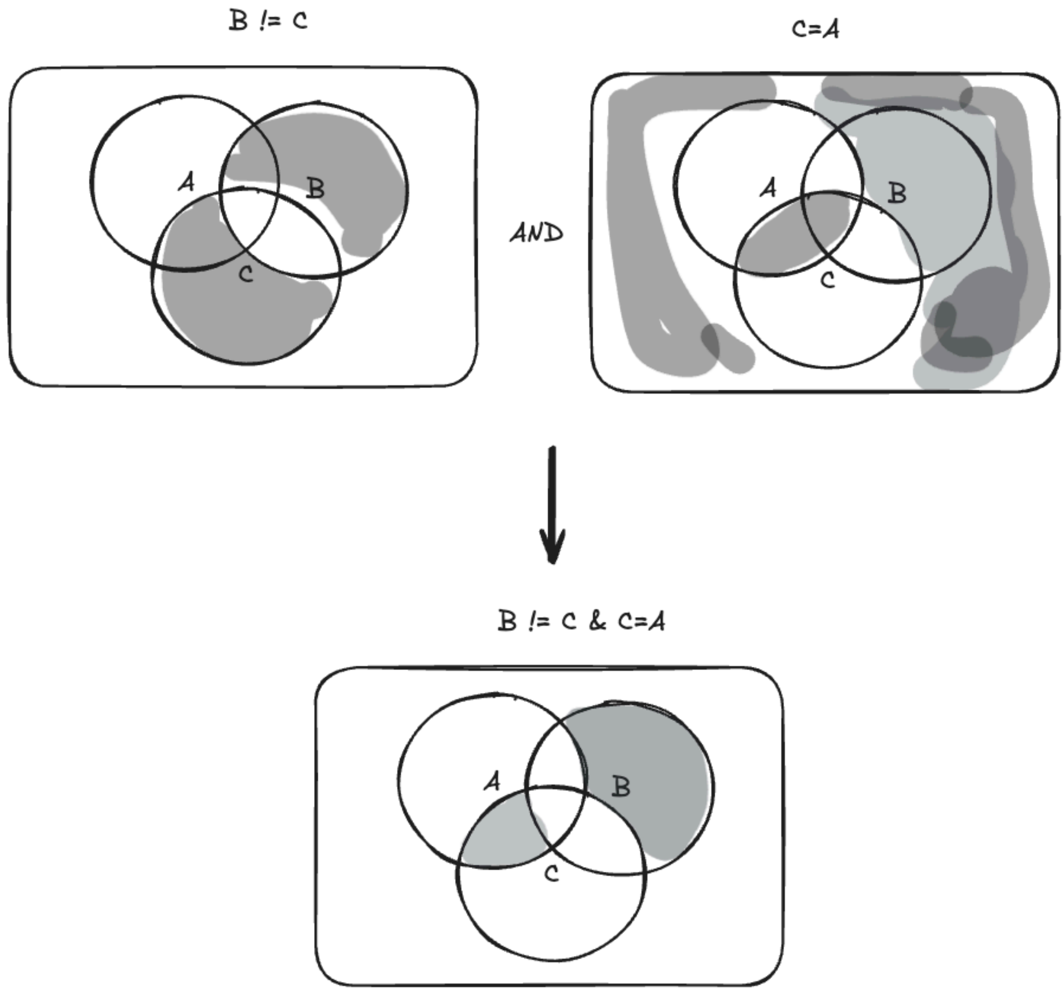
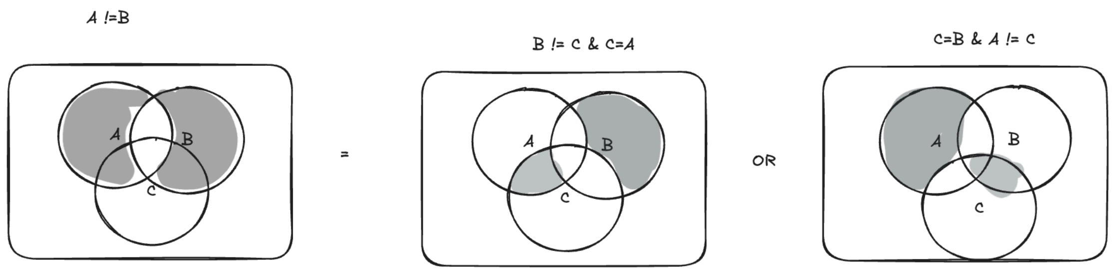


Figure 1: Venn diagram to show the overlap that creates $p(B = C \land C \neq A)$



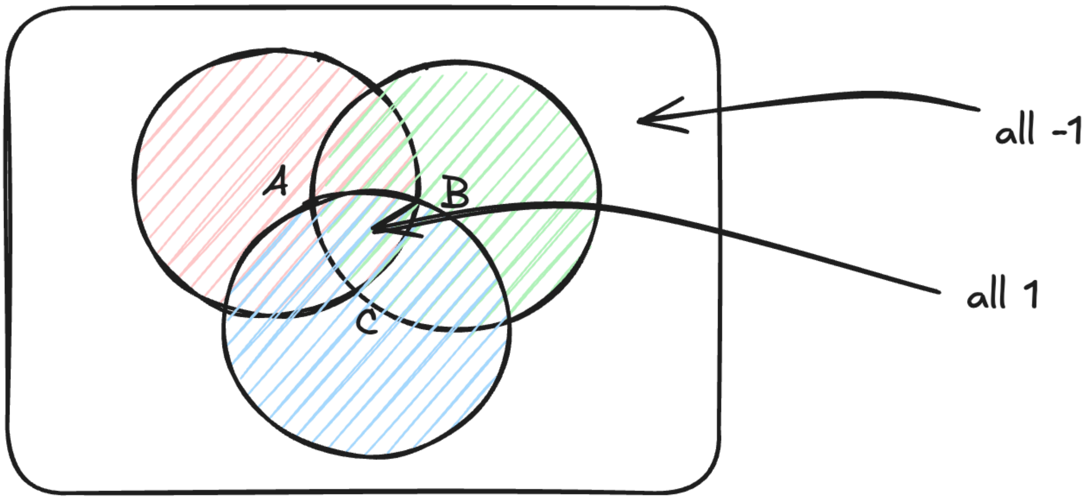


Figure 3: Saying sets of probabilities are non-contextual is equivalent to saying I can view them on a single Venn diagram.

We continue from Equation 1 by noticing that

$$\begin{aligned} p(B \neq C \wedge C = A) &\leq p(B \neq C) \\ p(C \neq A \wedge B = C) &\leq p(C \neq A) \end{aligned} \tag{2}$$

See Figure 4 for a visual interpretation

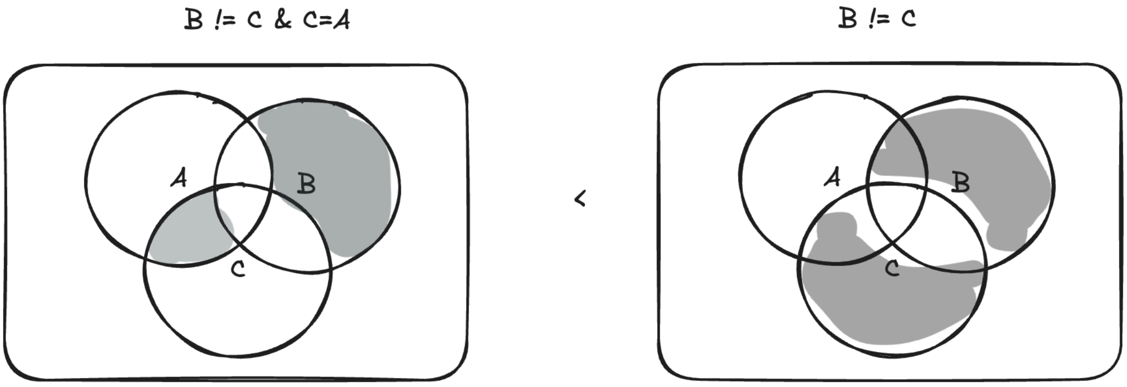


Figure 4: $p(B \neq C \wedge C = A) \leq p(B \neq C)$

combining Equation 1 and Equation 2 gives us back our Bell Inequalities:

$$p(A \neq B) \leq p(C \neq A) + p(C \neq B) \tag{3}$$

ibid for the other combinations of A,B,C.

Similarly, we can prove that:

$$p(A \neq B) + p(B \neq C) + p(C \neq A) \leq 2 \tag{4}$$

i.e if we have three binary variables, it is impossible to choose values such that they all differ from one another. Example: $A = 1$, $B = -1$, then $C = 1$ means it is the same as A but $C = -1$ means it is the same as B!

We have proved that the four Bell inequalities follow from non-contextuality. Therefore we have proved necessity. In fact the four Bell inequalities also imply non-contextuality. The sufficiency proof is not given but it exists.

5.2. CHSH-Bell Inequalities

We proved that non-contextuality implies the Bell inequalities and stated that this is both a *necessary and sufficient* condition. In fact we usually use 4 independent variables instead of 3.

5.2.1. Proof: Non-contextuality implies the Bell inequalities for CHSH version

Let us assign ± 1 to A, A', B, B'. For each such assignment we have:

$$AB + AB' + A'B - A'B' = A(B + B') + A'(B - B') \tag{5}$$

Since $B = \pm 1$: In the RHS of Equation 5, one bracket is always zero, so only one term ever contributes: $A(B + B')$ or $A'(B - B')$

Either $B = B'$ or $B = -B'$. If $B = B'$:

$$A(B + B') + A'(B - B') = A(B + B') = \pm 2A = \pm 2 \tag{6}$$

If instead $B = -B'$

$$A(B + B') + A'(B - B') = A(B - B') = \pm 2A = \pm 2 \tag{7}$$

Therefore there are only two possible values this expression can take:

$$AB' + AB' + A'B - A'B' = \begin{cases} +2 \\ -2 \end{cases} \tag{8}$$

This also implies:

$$-2 \leq AB' + AB' + A'B - A'B' \leq 2 \tag{9}$$

Although Equation 9 is a bit misleading because it implies that this combination can take other values between -2 and 2 such as 1 or 0.

The **assumption of decontextuality** enters into the following step: where we use the joint probability distribution $p(A, B, A', B')$ to take the following averages:

$$-2 \leq \langle AB' \rangle + \langle AB' \rangle + \langle A'B \rangle - \langle A'B' \rangle \leq 2 \tag{10}$$

Equation 10 represents one of the four CHSH inequalities. If Equation 10 holds, then a join probability distribution exists.

5.3. Aside: Compatibility and commutation

For two observables by distant observers Alice and Bob A, B the no-signalling theorem tells us that

$$\begin{gathered} [A, B] = 0 \\ [A', B] = 0 \\ [A, B'] = 0 \\ [A', B'] = 0 \end{gathered} \tag{11}$$

If the commutator is zero, two values are compatible.

❓ Conceptual Question

Why does the fact that measurements by Alice and Bob are compatible mean we *can* perform measurements which we observe marginal distributions like $p(A, B)$ etc ?

💡 Answer:

It is simply the fact that we need to have definite values for e.g A and B' for n measurements if we are going to construct probability distributions. This would not work if we could not measure both at the same time.

🔑 Take-Home Message

Experiments show violation of BI [4]. Therefore *experiments* show there are no non-contextual hidden variables.

5.4. Spin-1 Particles and Contextuality ⧖

❓ Conceptual Question

What is an exclusivity graph?

5.5. Dealing with Contextuality

5.5.1. Embracing Contextuality

Bohr, Heisenberg, and textbook QM do this

5.5.2. Explaining Contextuality

Bell, Bohm attempt this.

5.5.2.1. The firefly box: A *more contextual* example than QM

Suppose A, B, C binary and:

$$p(A \neq B) = p(B \neq C) = p(C \neq A) = 1 \tag{12}$$

a) This violates one of the Bell inequalities
b) This also leads to a contradiction - i.e assignment of values simultaneously to A, B, C is impossible.

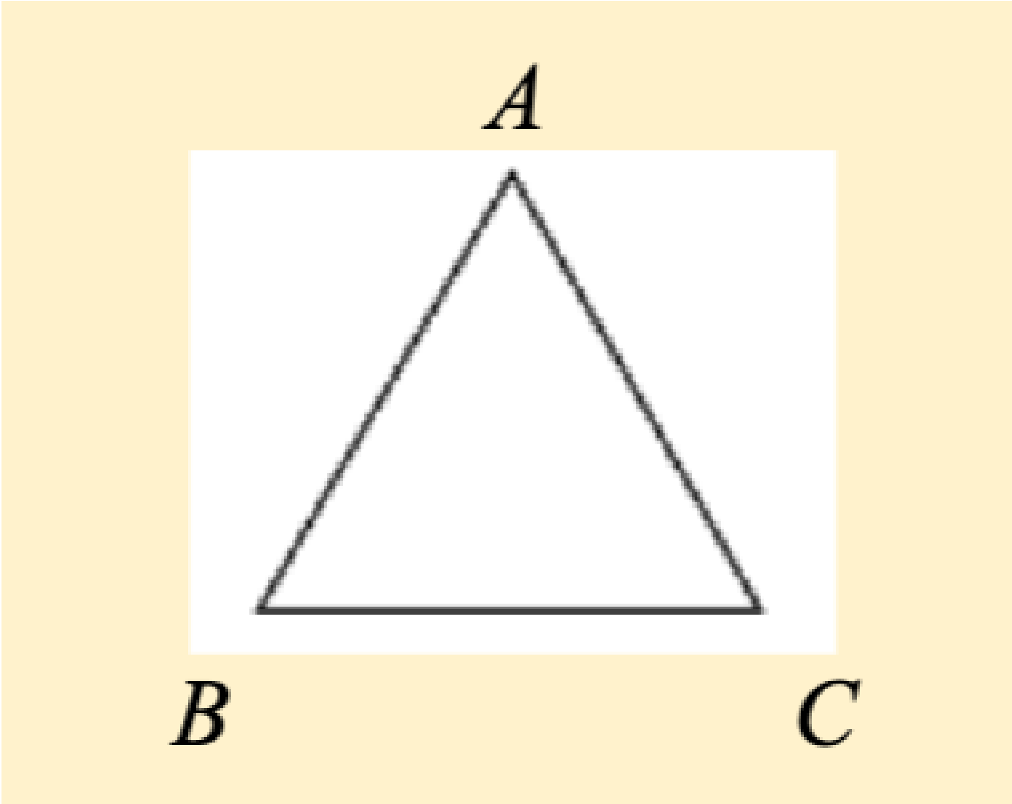


Figure 5: Exclusivity Graph. Edges connect mutually exclusive outcomes

But: we can think of a case like this in the real world.

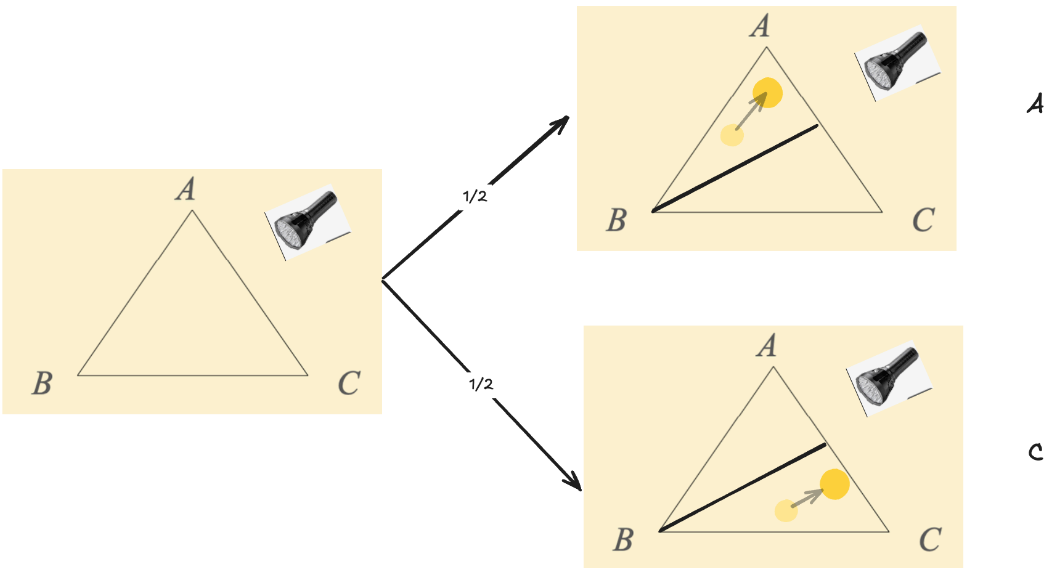


Figure 6: Firefly box. Firefly moves towards side where flashlight has shone. What we are really 'measuring' is therefore which *half* of the triangle the firefly was on when we make a measurement

For the firefly box, it does not appear contradictory that we cannot assign values A, B, C simultaneously because the act of measuring changes the outcome.

This is a *contextual hidden variables* explanation for the marginal probabilities $\frac{1}{2}$ that we measure for each variable A, B, C

Perhaps the measurements of momentum and spin in QM are equivalent to the measurements of “A, B, C” in the firefly box - i.e they can be explained by an underlying mechanism which leads to contextual sets of probabilities.

5.6. Eliminating contextuality requires retro-causality? Specker’s paper [2]

retrocausality the future can affect the past

measurement independence (MI) the choice of what to measure (AB, B C, or CA) is independent from the hidden variables λ. In the Specker Niniveh tail, the choice of what to measure depends on the hidden variable.

if A, B, C are binary “is the gem here?” variables, then from experience of the suitors, it seems like $A \to \neg B$ and $\neg B \to C$ but $A \to \neg C$. How do we explain this?

> **? Conceptual Question**
>
> “One day, however, the daughter jumps in and quickly opens a pair of boxes that had been predicted to be one full and one empty”
>
> So I am assuming here that the suitor said A B are both empty and she opens B C ? Because the rules of the game don’t allow the suitor to open two boxes which he believes are one full and one empty.
>
> The father, being the better prophet, would have placed one gem in B C.

> **? Conceptual Question**
>
> Is measurement independence really the same as no-retrocausation? In one, it seems like the choice affects the λ whereas in the other the λ affects the choice?

> **? Conceptual Question**
>
> Why is this retrocausality a violation of measurement independence? Is measurement independence a form of free will?

> **? Conceptual Question**
>
> The third box cannot be opened. Aside from **retrocausality**, it seems suspicious to base a theory on boxes that physically cannot be opened for some mundane reason.

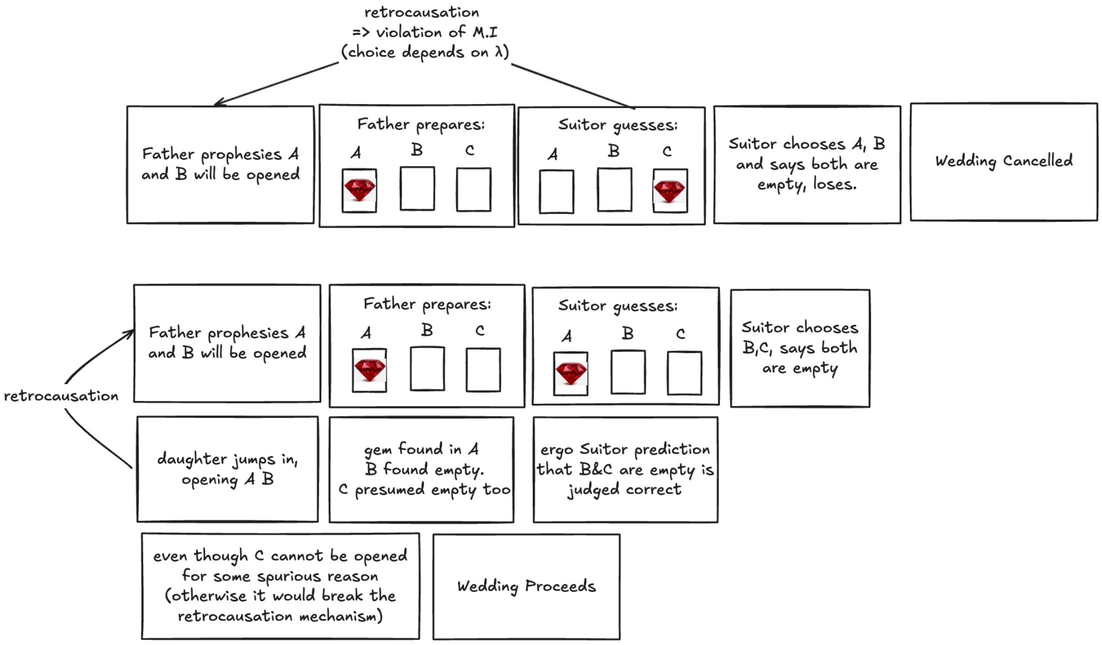


Figure 7: Fable of Niniveh

> ❓ **Conceptual Question**
>
> In the [2] the punchline is that is $A \to B$ does not mean $B \to C$, not sure how that maps on to the discussion here.

> ❓ **Conceptual Question**
>
> Why is **measurement independence** equivalent to retro-causality?

6. Lecture 5B: Contextuality and Non-locality: Four Proofs of Bell's Theorem

20.05.2025

We derive BI from locality assumptions (assuming contextuality) meaning that QM also shows there are no *local* extensions of QM.

The headline from this lecture is

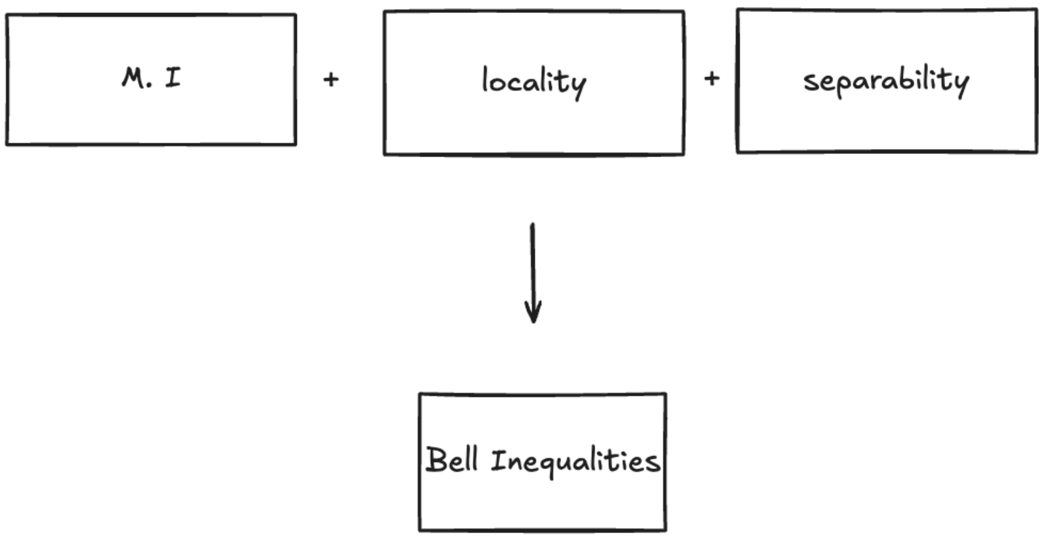


Figure 8: punchline of Lecture 5B

7. More Reading Notes

7.1. EPR [3]

The EPR argument essentially says that holding QM to be a complete theory leads to a contradiction.

reality criterion If, without in any way disturbing the system, we can predict with certainty (i.e with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.

Consider these statements:

A: ψ is incomplete
B: two non-commuting observables cannot have simultaneous reality

According to the QM formalism, two non-commuting observables cannot be simultaneously known. Therefore, either QM is incomplete, or two non-commuting observables cannot simultaneously be real:

$$A \lor B \tag{13}$$

If you assume ψ is complete $\neg(A)$, then we conclude that two observables *do* have simultaneous reality, if we take the criterion of reality as EPR do.

$$\neg(A) \rightarrow \neg(B) \tag{14}$$

But this leads to a contradiction, since if we accept the Quantum Formalism then $A \lor B$ must be true. We conclude that ψ is incomplete and that A is true.

7.2. Bell, Against Measurement

7.2.1. Notes

7.2.1.1. Question: What happens first, the LL jump or the vN-Dirac jump?

LL and vN-Dirac look at collapse differently. The LL jump is one of a 'classical' apparatus into an eigenstate of its 'reading'. LL uses this to argue for the Dirac-vN jump ("it follows from this that $A_n(q)$ is proportional to the wavefunction of the electron after the measurement" LL22). This seems to imply that the LL jump occurs first.

Dirac's explanation: "measurement, causes the system to jump into an eigenstate of the dyamical variable that is being measured" also implies that it is the measurement "jump" which causes the Dirac jump.

7.2.1.2. Question: Doesn't the very definition of eigenstates imply a shifty split?

7.2.1.3. Question: How can we ban the word measurement if it is inherent to the theory?

I don't really see how it is possible to define a basis in QM without referring to the results of 'measurement'. So I disagree with Bell that it would be clearer to say Ψ_n is the state in which an "experiment to ~~measure~~ ascertain" an observable X will always have the result n, would be any clearer.

8. Ramblings

8.1. "Solving" the measurement problem will give us a definition problem

Let us say that we have "solved the measurement problem" and described how unitary evolution can take a system+ apparatus, starting in a definite product state at $t = 0$ $\Psi_{\text{sys}}(0)\Psi_{\text{app}}(0)$, entangled at some $t \neq 0$, $\sum \alpha_{i,j}\varphi_{\text{sys},i}(t)\varphi_{\text{app},j}(t)$ where $\varphi_{\text{sys},i}(t)$ is a basis of the system and $\varphi_{\text{app},j}(t)$ is a basis of the apparatus, to evolve into a state where the apparatus has a definite reading ($\varphi_{\text{sys},m}(t)\varphi_{\text{app},n}$), or even ($\varphi_{\text{sys},m}(t)\sum_k \varphi_{\text{app},k}$) where the sum is over semi-classical wavefunctions of the apparatus which all have the same definite reading, corresponding to the observable m in the system). Then, we have removed 'measurement' as a primitive in our QM formalism. But it seems like we need to remove eigenstates? Eigenstates φ_m of our system are defined *in terms of results of measurements.* If we have succeeded in our proof, we are left scratching our heads as to what this eigenstate represents? Let's say, naively, that it represents the state of the system, upon being measured, having a definite value for the observable (namely m). That phrase, "upon being measured", is shorthand for the hypothetical proof from the TDSE we have given above. But this hypothetical proof relies on Ψ_m being well-defined, and hence it still relies on the primitiveness of measurement. We are then left scratching our heads - if we have explained "measurement", if it is no longer a primitive, and we have used a basis φ_m to do so, then surely we should understand the basis φ_m as something other than "states in which the result of measurement are certain".

- LL collapse: the measuring apparatus is posited to have a well-defined **macroscopic** state. This implies that a 'Dirac-von-Neumann' style of the wavefunction has occurred (or will occur? What comes first?).
- Dirac-vN collapse is the wavefunction actually 'jumping' to one of its eigenstates.

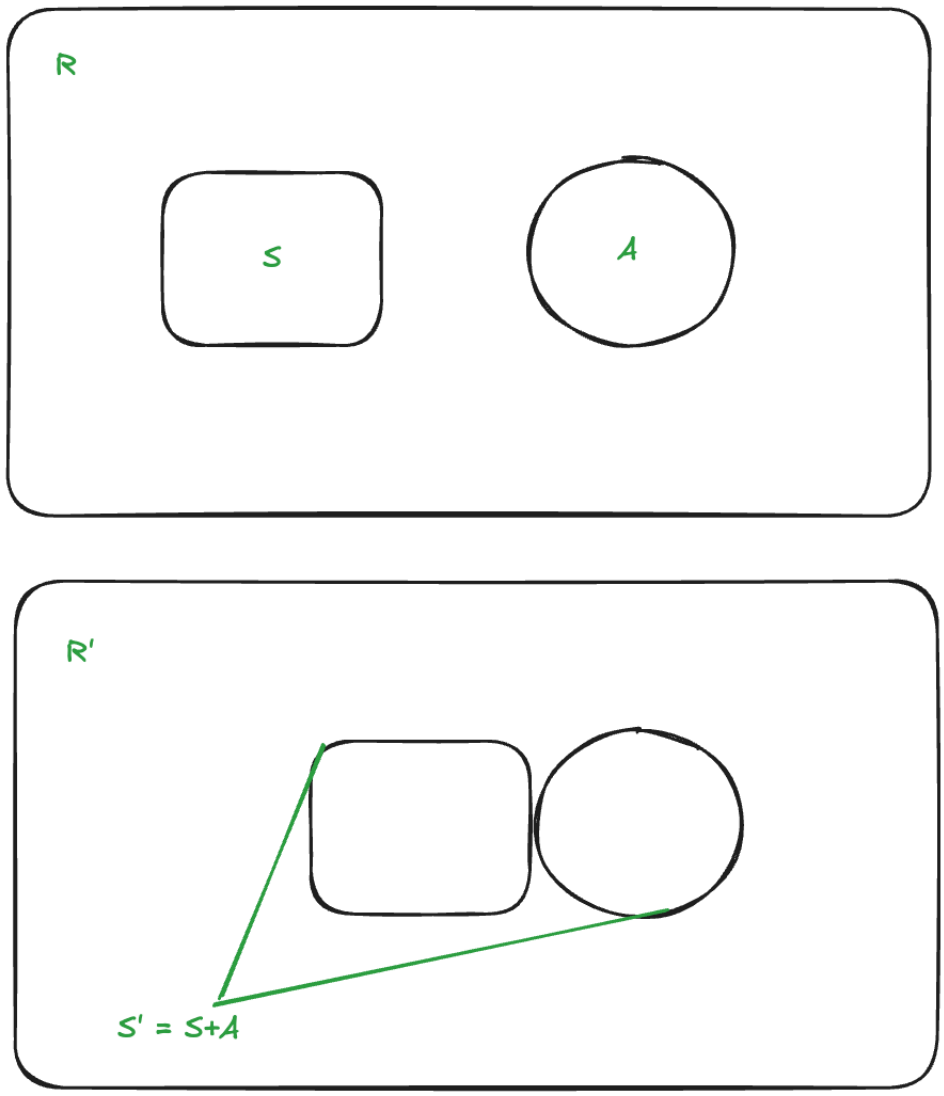


Figure 9: Treating the measurement apparatus as classical (top) vs quantum (bottom).

Related reading:

- 10 theorems [5]

9. EPR (Stern-Gerlach version)

EPR infers determinism from the experiment, but holds **locality** sacred [6].

a) either a signal is travelling faster than the speed of light between the two particles when one of them is measured or

b) some property, unmodelled in QM, determines the outcome. This is a "real" property because we know with certainty what it is before measuring it.

c) The act of measurement at A modifies the particle at B (this is basically (a))

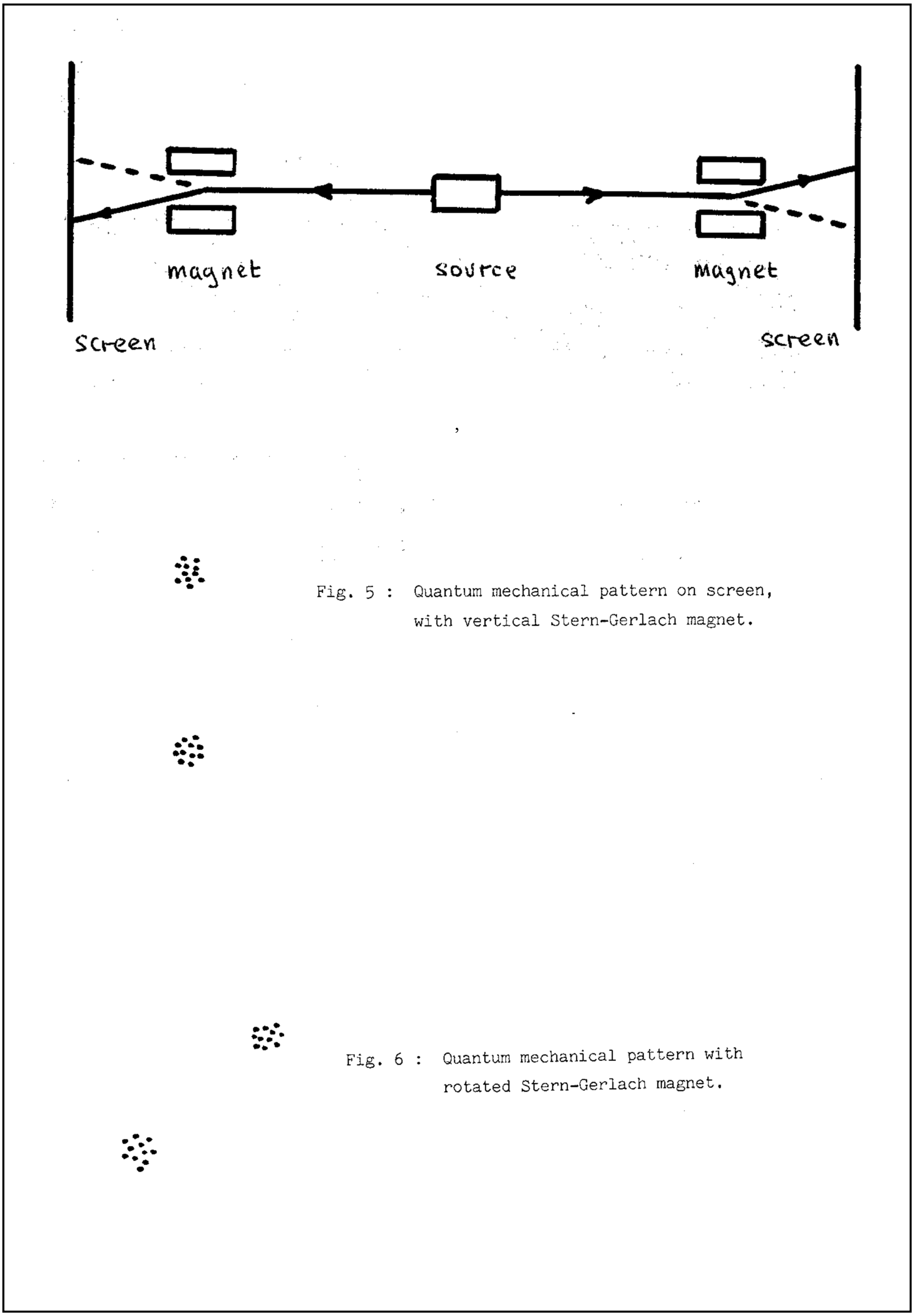


Figure 10: Figs taken from Bell's 1980 article [6]

EPR correlations of a singlet pair beg an explanation.

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