DRAFT
P3 -group 7

Isaac Newton's Infinite Series for the Sine

Emma Ottenhof and Victor Elgersma

14 May 2026
presentation due : 9th of June
report due: 16th of June

Contents

Glossary

Moment - equivalent to an ordinate or (quantity on the y axis), but with an emphasis on motion. A Moment appears to always be a function of time.
Superficies - The Area

Question

1. What does N. mean by 'Moment'?
2. What does N. mean by 'Superficie'?

Introduction

We provide a line-by-line analysis and modern re-interpretation of a section on the infinite series for the sine, taken from an English translation of Newton's *Analysis by means of Equations with an infinite number of terms*, first published in Latin in 1711.

We feel that Newton's diagrams, in which he presents his argument, suffer from the use of too many letters. These letters distract the modern reader from the clarity of his argument. Therefore in addition to a line-by-line commentary we have provided a streamlined modern-recasting, in which we will use letters sparingly. However, since Newton uses both the latin and greek alphabet in his exposition, we will adopt a (reduced) version of the cyrillic alphabet in our modern re-casting (Б, Г, Ж…).

Letters common to the cyrillic and latin alphabets should be understood to be latin letters, and therefore part of Newton's original diagram. This should make conceptual discussions of Newton's argument simpler, as it will always be clear when we are talking about Newton's reasoning and when we are referring to our own understanding of Newton's concepts.

Where we use Latin and Greek letters (A,B,C), (α, β), they should be understood to refer to elements in Newton's diagram as he wrote them.

Our recasting will contain some objects (such as x, y) in common with Newton's presentation.

Line by line analysis of Newton sine series

> *The Application of what has been ſaid to other Problems of that Kind.*

> 37. Let ABD be any Curve, and AHKB a Rectangle, whoſe Side AH or BK is Unity:

When considering areas under Curves Newton prefers to consider a 2-dimensional x-axis with a side length unity[1]

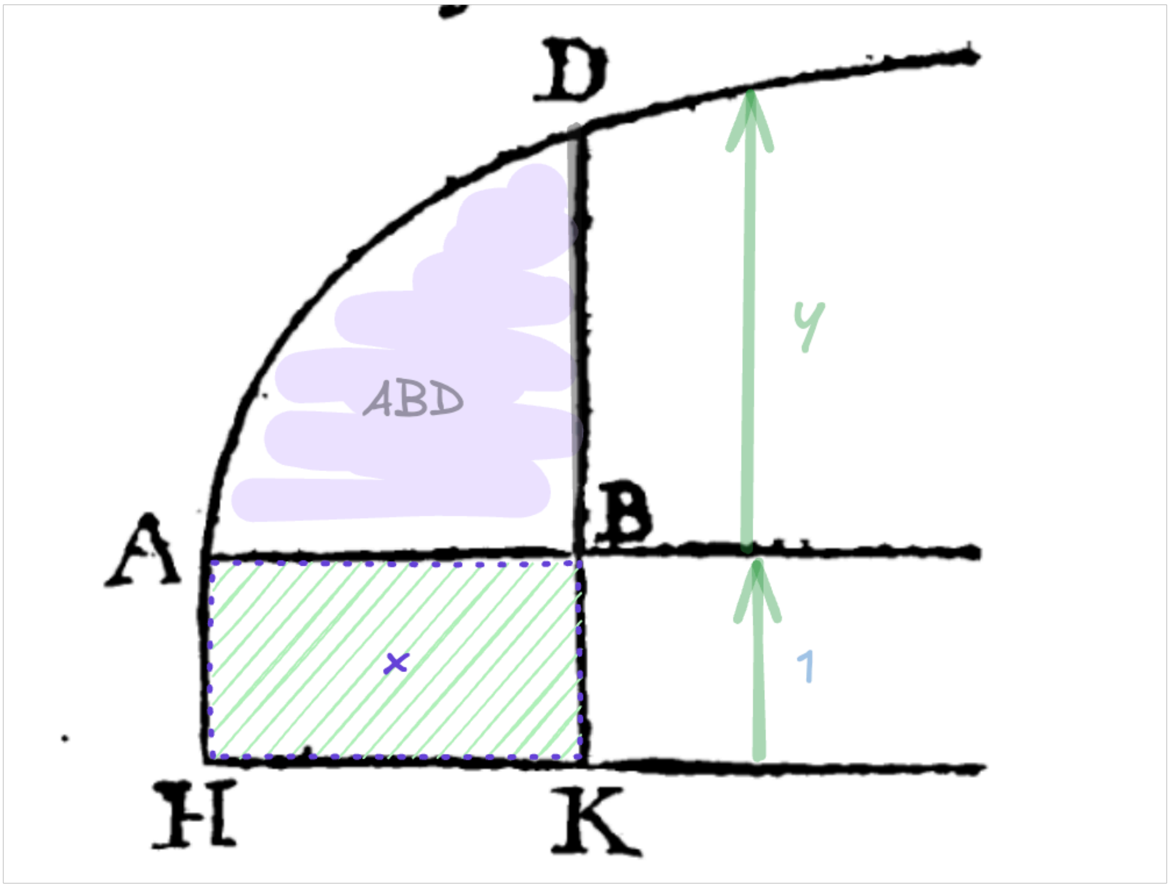
> And imagine the Right Line DBK to move uniformly from AH, ſo as to deſcribe the Areas ABD and AK; and that BK (1) is the

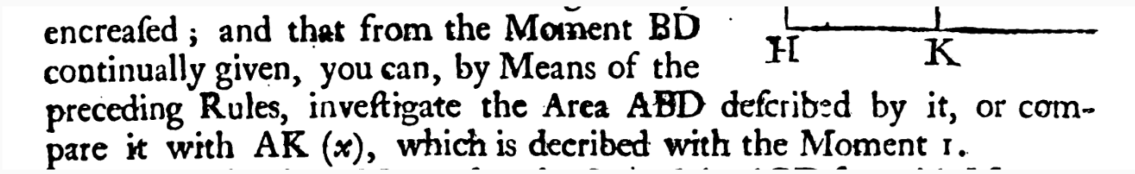
> Areas ABD and AK; and that BK (1) is the Moment with which AK (x), and BD (y) the Moment with which ABD is gradually encreaſed; and that from the Moment BD

It appears that by "Moment" Newton is referring to a value Ж such that the respective areas increase by Жdx.

It is clear from our diagram that x is supposed to have the dimensionality of area, whereas y is supposed to have the dimensionality of length. However, we don't believe that it is merely an idiosyncrasy that Newton wants his x-axes to be two dimensional. As we can see from our diagram, there is an obvious parallel between increasing the area of the rectangle by the 'Moment' 1 and increasing the area of the arbitrary curve by the 'Moment' y.

[1]We also saw this in Newton's Treatise of the Quadrature of Curves in presentation P1-8





> encreaſed; and that from the Moment BD continually given, you can, by Means of the preceding Rules, inveſtigate the Area ABD deſcribed by it, or compare it with AK (x), which is decribed with the Moment 1.

AK is really shorthand for the area $ABKH$. We assume that the 'preceding rules' are the rules of integration and infinite series. When Newton says "investigate the Area ABD described by it, or *compare* it with $AK(x)$", the most reasonable interpretation is that he wants to draw attention to how the area under the curve changes *as a function* of x. Therefore, in the following analysis, we will call the area Д(x).

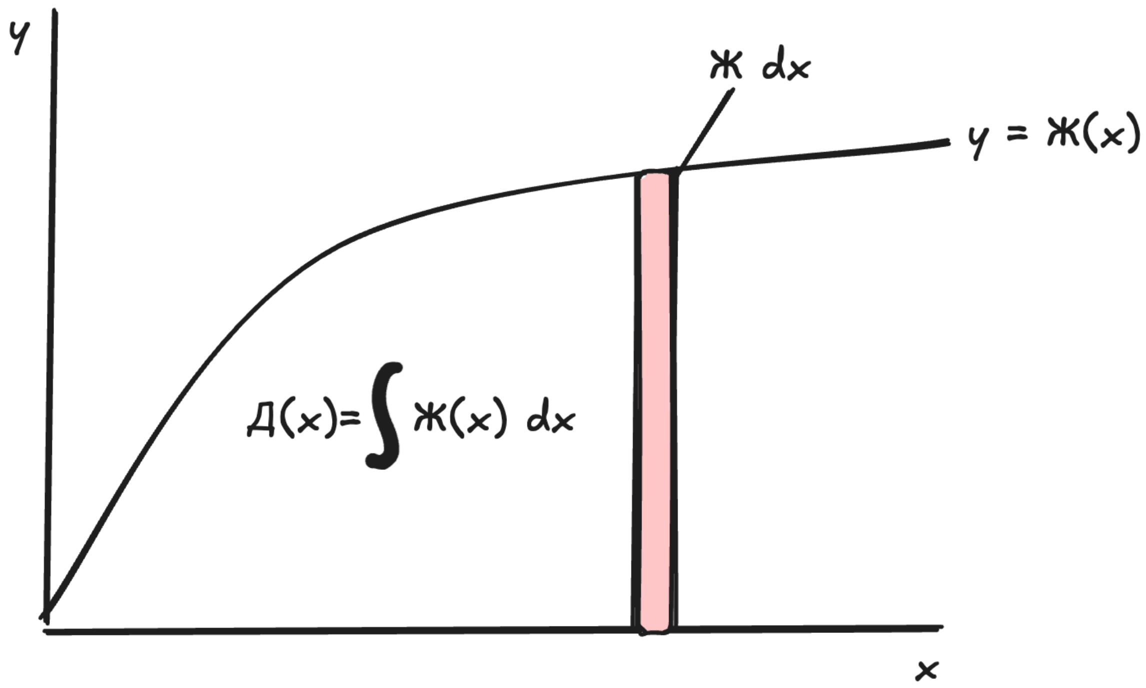
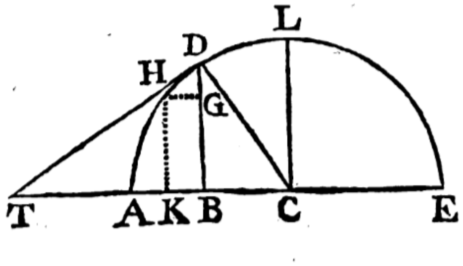


Figure 1: We want to find out how taking x to $x + dx$ will change the area $Д(x)$. The area $Д(x)$ is increased by the moment Ж to $Д(x) + Жdx$

> Now by the ſame Means that the Superficies ABD from it's Moment being at all Times given, is diſcovered by the foregoing Rules, by the like Means may any other Quantity be inveſtigated from it's Moment given in like manner. The Thing will be clearer by an Example.

In the above passage, Newton tells us that if we know $Ж(x)$ at all times, then we will know $Д(x) = \int Ж(x) dx$ at all times.

It is striking that Newton still uses an x instead of a t for his independent variable, as it is clear from this discussion that the Moment should be viewed as a function of time.

> “
> *To find the Lengths of Curves.*
>
> 38. Let ADLE be a Circle, the Length of whoſe Arch AD is to be inveſtigated. Draw the Tangent DHT, and having compleated the indefinitely ſmall Rectangle HGBK, and put AE $= 1 = 2$AC, it ſhall be as BK or GH the Moment of the Baſe AB (x) to HD the Moment of the Arch AD : : BT : DT : : BD ($\sqrt{x-xx}$) : DC ($\frac{1}{2}$) : : 1 (BK) : $\frac{1}{2\sqrt{x-xx}}$ (DH). And ſo
>
> 
>

Bibliography

Newton. (1711). *The Application of what has been said to other Problems of the Kind.* https://nx89456.your-storageshare.de/s/Ag2xzMJ8joDXkLd