Critical Question: Does the decoherence argument abuse the redundancy of ρ?

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613 words

Improper mixtures and proper mixtures can have the exact same density matrix. I suspect KG's attempt to derive collapse and the Born rule by the elimination of coherence **abuses this fact**, by inferring that states / proper mixtures with the same density matrix are the same. Is this correct?

The joint $S + A$ system ends in the state $\Psi = \sum c_n \Psi_n$ after a measurement, where the Ψ_n each have definite apparatus pointer readings. I am assuming this is an entangled state where each $\Psi_n = \Psi_{\text{app, n}} \Psi_{\text{sys, n}}$ so if the apparatus has reading g_n the apparatus will be in state $\Psi_{\text{app, n}}$ and the system in state $\Psi_{\text{sys, n}}$. This is just the assumption that our apparatus is 'ideal', so the product state $\Psi(0) = \Psi(0)_{\text{app}} \Psi(0)_{\text{sys, n}}$ will evolve with certainty into $\Psi = \Psi_{\text{app, n}} \Psi_{\text{sys, n}}$ I don't think it means, as Bell implies, that "KG tacitly assumes the Dirac rules at S'/R' in order to to get correlations between successive (moral) measurements" (p. 37). Also not entirely clear what "moral" means here. So, at this stage of the argument we are in this (unusual in normal QM) situation where a 'measurement' has occurred but what we call 'collapse' hasn't happened yet. And the wavefunction describing $S + A$ is still in this superposition $\sum_n c_n \Psi_n$

Rather than applying 'collapse' directly at the modified S' / R' boundary, KG infers the *appearance of collapse* and the Born rule from the following argument:

The density matrix is introduced:

$$\rho = \sum_n \sum_m c_n^* c_m^* \Psi_n \Psi_m^* \tag{1}$$

This 'matrix' is really just a function of two position vectors $\boldsymbol{x}$ and $\boldsymbol{x}'$ (I am not sure in what sense it is a 'matrix', actually)

$$\rho(\boldsymbol{x}, \boldsymbol{x}') = \sum_n \sum_m c_n^* c_m^* \Psi_n(\boldsymbol{x}) \Psi_m^*(\boldsymbol{x}') \tag{2}$$

Why is this the density matrix? I offer an explanation below:

> The density *operator* is given by:
>
> $$\rho = \sum_i p_i |i\rangle\langle i| \tag{3}$$
>
> Where we take the S+A system to be in one of the definite states $|i\rangle$, assigning a subjective probability p_i to each of them. But of course, in this case we *know* exactly what state our system $A + S$ is in, so there is no subjective probability:
>
> $$\begin{aligned} \rho = \sum_i p_i |i\rangle\langle i| &= |\Psi\rangle\langle\Psi| + 0 + 0 + ... \\ &= |\Psi\rangle\langle\Psi| \end{aligned} \tag{4}$$
>
> The corresponding 'matrix', in the position representation, is:
>
> $$\langle \boldsymbol{x}|\rho|\boldsymbol{x}'\rangle = \langle \boldsymbol{x}|\Psi\rangle\langle\Psi|\boldsymbol{x}'\rangle = \sum_n \sum_m c_n^* c_m^* \Psi_n \Psi_m^* \tag{5}$$
>
> Where the last equality just follows from $\Psi = \sum_m c_n \Psi_m$.
>
> QED.

Now for the 'abuse'. If we consider the non-trivial density operator associated with a classical apparatus which is in a state i about which we are ignorant (but can assign probabilities p_i), then:

$$\hat{\rho} = \sum_i p_i |i\rangle\langle i| \tag{6}$$

whose matrix elements are

$$\langle \boldsymbol{x}|\hat{\rho}|\boldsymbol{x}'\rangle = \sum_i p_i \langle \boldsymbol{x}|i\rangle\langle i|\boldsymbol{x}'\rangle = \sum_i c_i c_i^* \Psi_i \Psi_i^* \tag{7}$$

Now, looking at (5), if we can make the cross-terms disappear, we can turn it into (7). The 'decoherence argument' is to claim that, since these cross-terms do not appear in the expectation values of all observables A, we cannot distinguish between ρ and $\hat{\rho}$ matrices, and we are justified in replacing the 'state' of the system with the proper mixture with density operator (7).

So the argument seems to rely on the unfortunate truth that there is a lot of redundancy in $\langle \boldsymbol{x}|\hat{\rho}|\boldsymbol{x}'\rangle$ - it can represent both pure entangled states and proper mixtures between states.

It is convincing enough to me that we can act as if Ψ *had* collapsed onto Ψ_m for the purposes of calculating expectation values - not that this collapse actually occurs. Surely the TDSE cannot turn an entangled state into a proper mixture because it is reversible and cannot destroy information?

For me, it seems that the fact that this this density operator *appears just like* a classical system where we are simply ignorant of the exact state of the system points to a fundamental entropy-destroying, indeterministic, 'collapse' process happening at the wave-function level. If the TDSE *is* deterministic and doesn't destroy information, Bell claims this points to hidden variables:

> "In particular the readings of experimental apparatus are supposed to be really there before they are read [...] if the exactness of the Schrodinger equation is maintained, I see this leading towards the picture of de Broglie and Bohm" (p.37-p.38)

But if 'hidden variables' deterministically decide which branch of the wave function survives, how do we get the irreversibility necessary to explain entropy?